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Appendix to a Memoir on the Mississippi, No. XXX. of the 1st part of this Volume.—By William Dunbar, of the Natchez, communicated by the Author, through the President of the Society.

Read October 5th, 1804.

ALTHOUGH the memoir was not intended to convey opinions upon the theory of rivers, yet as it contains observations and remarks, which are at variance with the doctrines delivered to us by several of the most eminent mathematicians of Europe, it may seem that a short apology is necessary.

This subject has been treated by mathematicians of the first order in Italy, France and Germany, but more especially the former; and generally such partial views only have been taken of the subject, as have furnished them with the amusement of an elegant application of calculus. The theorems of Guglielmini have been held in the highest estimation, and, perhaps unfortunately for the progress of science, prevail too generally at this day. The theory of spouting fluids issuing from orifices with velocities in the ratio of the square-roots of the respective columns, has been applied without modification to every motion of water. Mariotte, Varignon and Guglielmini have made it the basis of complete systems of hydraulicks. Varignon has composed many analytical memoirs upon this theory; and Gravesande, Musschenbroek and Belidor deliver no other principles: Guglielmini has (in addition to this theory) introduced something not very intelligible on the energy of deep waters, which he considers as the cause that rivers are not stagnant at their mouths, where there is, as he supposes, no declivity of surface.

Theories formed by ingenious men, without any regard to experiment, have too frequently led their authors into absurdities, and it is surprising that a theory so contrary to fact in the most familiar and obvious circumstances, should have met with so much attention: to defend it must involve its advocates in an inextricable dilemma: it results from this theory, that at one foot under the surface of the most sluggish stream, there exists a current at the rate of 8 feet per second ($5\frac{1}{2}$ miles per hour) exceeding that at the surface; so extraordinary a case must have been long since familiar to boatmen, but it is well known that if a person on board of a boat floating down the stream, thrust his hand and arm at full length under the surface, he will find the water (relatively) as still as a mill-pond; it cannot be supposed that river-navigators would have so long neglected to take advantage of so favorable a circumstance; oars and sails would have ceased to be necessary for descending great rivers; the velocity (from theory) at the small depth only of about 16 feet below the surface, exceeding that at the surface 32 feet per second, if we permit a drag of proper construction to sink to that depth, connected with a vessel, she would be drawn along with a velocity, exceeding that at the surface about 22 miles per hour. Again, however minute the velocity may be at the surface, that at the bottom of a deep river would be immensely great: what shall we think of that of the gulf stream? or even of the Mississippi, where the depth is supposed to be 50 fathoms, and which would produce 140 feet per second, little short of one hundred miles per hour? now as it is known that a velocity of 3 inches per second will just begin to work upon clay, and that of 3 feet will sweep along shivery angular stones of the size of an egg, and as according to our theory, the evil ought to be perpetually upon the encrease, in as much as the velocity augments with the depth, it must have resulted that by such incredible velocities as are deducible from the theory, the bowels of the earth must have been long since torn up, and this globe have been no longer a fit habitation for man: a system so pregnant with consequences contradictory to the order and regularity which are the result of the laws of nature, must be abandoned. Without the aid of philosophy it must have been remarked by every common observer, that the most furious torrent (directed into a new channel) after breaking up and tearing every thing before it, does at length fashion its own bed, in respect to breadth and depth, so as to be perfectly adapted to the momentum of its waters; it is no longer a furious torrent, but a mild placid stream. Nature aims continually at an equilibrium; in rivers which have for a course of ages occupied the same channel, the accelerations and resistances are so perfectly counterpoised, that a complete equability of current takes place for a great extent (i. e.) so far as the regimen of the river has established itself; abrasion at the bottom of the river ceases; this can only be the consequence of reduced velocity, contrary to our theory, which demands velocity encreasing with the square-root of the depth. Mathematicians and engineers who have calculated upon so false a theory have been most egregiously disappointed in their expectations; a canal was projected to supply the city of Edinburg with water, the celebrated M'Laurin calculated the quantity it ought to deliver, and the no less celebrated Desaguliers who was to conduct the enterprise, and whose theoretick principles were somewhat corrected by experimental knowledge, reduced to nearly one half the calculation of the former; the work was executed to the satisfaction of both, and the result was, that the actual quantity of water delivered was $\frac{1}{11}$ of that calculated by M'Laurin and $\frac{1}{6}$ of that of Desaguliers.

The great improver of the Steam-Engine, Mr. Watt, informs us, that a canal of 18 feet wide at the surface, 7 feet at bottom and 4 feet deep, runs with a velocity of 17 inches per second at the surface, 10 at the bottom, and 14 in the middle; according to the theory, the velocity at bottom ought to have been 16+17 or 33 inches in place of 10, abating the effect of friction upon the bottom of the canal.

A very few persons have thrown light on this subject by some valuable experiments, none have been more successful than the Chevalier Buat: aided by St. Honore, a young officer of Engineers, he has adapted analytical forms of expression conformable to the operations of nature. Buat measured velocities at the surface and bottom of canals and rivers, and has discovered the following laws. "In small velocities there "is great disproportion between the surface and bottom; and "in very great velocities, the ratio approaches to equality; in "general the following rule will solve the problem: Take unity "from the square-root of the superficial velocity per second "expressed in inches, the square of the remainder is the veloci-"ty at bottom." Thus a velocity of one inch at the surface will give no sensible velocity at bottom, but a velocity of 36 inches at the surface, will give 25 inches at bottom; Watt's canal corresponds with this law, and it is probable that the law holds good in all artificial canals and rivers of moderate depth; but in great and deep rivers, whose regimen is established, there is great reason to believe that the velocities at bottom are much less than would result from Buat's rule, because as has already been observed, that so far from abrasion taking effect at the bottom of such rivers, they are actually rising by a slow progress, which is regulated by the protrusion of the cradle of the river into the ocean. Many more arguments from fact might be drawn in opposition to this theory; I shall only observe that it is known to fishermen, that the migration of fishes is performed near the bottom of rivers against the stream, and in descending they almost float upon the surface; a curious account of the latter is given by Bartram in his account of St. John's river in East Florida.

We shall now endeavour to shew that the theory is unphilosophical and contrary to hydrostatical laws.

Let A B (Plate V. Fig. 1.) represent the longitudinal section of a river flowing with uniform velocity from surface to bottom, and let us enquire what change ought to take place in the velocity at different depths, caused by the pressure of the fluid: Let us suppose a wall C D, forming a complete transverse section of the river, and moving uniformly with the current from A, to B, and that the whole inferior part B, is instantaneously removed; if now orifices be made in the wall at 1, and 2, the water will flow out in the direction of the stream, with velocities in the ratio of the square-roots of the columns above the respective orifices; upon this partial view of the subject, the theorists have built their system. Again, supposing all things to remain as before, the portion of the river B, being replaced, let us now suppose the superior portion A, to be removed, while the wall moves on uniformly with the current and portion B, if now the same orifices 1, and 2, be opened, the water will flow out with the same velocities as before, but in contrary directions, against the course of the river; hence it appears that the simple pressure of the water is equally disposed to produce inferior currents in any direction, the instant the equilibrium is destroyed; but it is certainly very unphilosophical to assert that the column 3, 4, will produce an increased current in the direction of the stream, while it is opposed by a column of equal pressure 1, 2: it cannot be asserted that any inequality of pressure, arising from the gentle declivity of the surface of large rivers, can produce any sensible effect; for should it be said that the pressing and opposing columns are not to be measured in contiguity to each other, but that the opposing column will be null, in consideration that a point is to be found on the surface of any river, upon the same level with any given depth higher up the stream; we reply, that this effect is totally destroyed by other concomitant circumstances. Great rivers whose regimen is long established flow with a very gentle declivity, perhaps in some cases not more than 2 inches per mile, but let it be supposed one foot; according to the theory the velocity of an inferior current a b, (Fig. 2.) at the depth of 16 feet a c, ought to be 32 feet per second, because at the point b, 16 miles below c, there is no opposing column: this is certainly the most favourable view in which the theory can be presented, but will not avail its advocates; for it cannot be shewn that the vis inertiæ of 16 miles of fluid can be overcome by a pressure of 16 feet, with the energy required by the theory; on the contrary, it is shewn by the experiments of M. Couplet at Versailles, that water conveyed in a smooth horizontal tube of 18 inches diameter and 43,200 inches in length, from a reservoir 12 feet high, issued with a velocity of less than 40 inches per second, (i. e.) less than $\frac{1}{8}$ of the velocity deduced from the theory; hence we see that the vis inertiæ of 43,200 inches of horizontal water combined with the friction of a tube 18 French inches in diameter, destroys $\frac{7}{8}$ of the velocity which the theory calls for; and if we should concede (what the theorist cannot demand) that of those $\frac{7}{8}$, $\frac{5}{8}$ are occasioned by the friction of so large a tube, and only $\frac{2}{8}$ left for the vis inertiæ of the water, and that it be allowed that every succeeding 43,200 inches destroy $\frac{1}{4}$ of the respective remaining velocities, we shall find that at the end of the 16 miles, the velocity of the issuing fluid will be less than $\frac{1}{2}$ inch per second. Were we to suppose a horizontal pressure at a, derived from a head of water e f, proportioned to the column f e, it is yet inconceivable that this should produce a continued velocity in the direction a b; water like all other bodies, when in a state of compression, will escape on the side of the least resistance, and in place of producing a current in the direction a b, against the vis inertiæ of 16 miles of fluid, will escape by the shortest passage to the surface, and bubble up at d, where it will form an elevation and encrease the superficial velocity. Were we disposed to suppress these arguments, and concede to the theorists the doctrine they have endeavoured to establish, the consequence would be equally ruinous of their system: let us therefore suppose that a current is produced from a, to b, with a velocity of 32 feet per second greater than at c; by a parity of reasoning it will at g be 64 feet greater than at c, and so in continuation gaining at the rate of 2 feet per second every mile; hence a river running one hundred miles, after it had gained the depth of 16 feet would run with a superficial velocity of more than 200 feet per second: had we assumed the depth of one foot only in the place of 16, it will be found from the above mode of reasoning, that the superficial velocity gained would be at the rate of 8 feet per mile: it is unnecessary to advance any thing further against a theory capable of yielding results so contradictory among themselves, and so totally at variance with fact and observations.

From what has been said we may conclude that the natural movement of fluids depends solely upon the declivity of the surface; the obstructions arising from friction, adhesion and vicidity, being greatest at the bottom and sides, the velocity of the current will consequently be greatest at the surface and in the middle of the channel where there is no deflecting cause. Buat observes, we may be assured that the central filament of water running through an inclined cylindric glass tube flows with the greatest velocity, it being known that however smooth and polished the interior surface of the tube may be, the retardations from friction are very considerable; if we suppose the superior half of the cylinder to be removed with its included water, the relative velocities of the inferior half will continue the same, and he sees no reason to doubt that all rivers and canals move upon the same principles. We shall consider this object in another point of view, leading to the same conclusion. Let the solid A B, (Fig. 3.) of indefinite length, be divided into a number of very thin and polished laminæ, and placed upon the inclined plane B C, with such declivity as that the solid may just begin to move by the power of gravity down the inclined plane from B, towards D, when the lamina 1, shall have gotten into the position I, the lamina 2, possessing a greater facility of motion over 1, than this last has over the inclined plane, will have also made one step over 1, and will be found in the position II; in like manner, the lamina 3, will at the same instant move over the lamina 2, and make one step beyond it and will be found in the position III, and so of all the other superior laminæ which will be found respectively in the situations represented in the figure. Water being composed of parts possessing extreme mobility, it is not unreasonable to conclude that its motion along an inclined plane, will be somewhat analagous to that of polished laminæ, but as fluids press laterally as well as perpendicularly, there must be corresponding retardations at the sides as well as at the bottoms of rivers and canals.

The energy of deep rivers which has been insisted upon by Guglielmini is not entitled to much notice: we must however admit that water, like solid gravitating bodies descending along an inclined plane, will acquire velocity until the accelerations and resistances are in equilibrio, but from its extreme mobility is more liable to lose it: a globe of solid matter rolling along a horizontal plane loses its motion instantaneously on its falling to pieces; it is not therefore astonishing that water, divisible into the minutest parts, descending into every cavity and deflected by the smallest obstacles, should be speedily deprived of its velocity. As a small body impinging with great velocity upon a large mass may communicate no sensible velocity to the compound, so in like manner, a descending torrent being received into a more capacious bed is totally disarmed of its fury and moves on with a new velocity proportioned to the new declivity.

Deep rivers moving with a certain velocity and meeting with obstacles will exert the energy spoken of by Guglielmini, that is, like all other heavy bodies in motion, they will endeavour to persevere in the right line of their last motion, and the waters will accumulate against the object, having a tendency to rise to the height of a reservoir, which would produce the actual velocity of the current: thus if M. Pitot's tube A B, (Fig. 4,) be set with its horizontal orifice B, against the current, the water will ascend to C, a height proportioned to the velocity of the current at B; that is, the column C D, pressing above an orifice in any reservoir would produce a velocity in the spouting fluid equal to that of the river at B: this instrument may be commodiously used for ascertaining the velocity of currents where great accuracy is not required, and in low velocities; the tube might be graduated so as to give the velocity by inspection: it may also be used to determine the difference of superficial and inferior currents. Were the theory true which we oppose, a remarkable effect would be seen in Pitot's tube; the water ought to rise in the tube to a height above the surface of the river, equal to the depth at which it is plunged below the surface, and if the tube be rendered stationary the water will rise still higher by an additional height corresponding to the superficial velocity: thus Pitot's tube placed at the depth of 16 feet in a river whose superficial velocity is 8 feet per second, would raise the water to the height of 17 feet above the surface of the river, and orifices being made in the side of the vertical tube, the water would flow out with various velocities depending on the position of the respective orifices. What a discovery this for raising of water without machinery!! however absurd this result may appear, it is fairly deducible from the theory.

In any great river, water flowing in the direction 1, 2, 3, (Fig. 5.) and impinging against the bank at 3, will there accumulate and rise higher than at 4 (which is always lower than at 2,) if the velocity of the current be 8 feet per second, it will have a tendency to rise one foot, but from the unconfined state of the water, a considerable abatement will take place; the water accumulated at 3, is the cause of all eddies; it falls off in all directions from the thread of the current, producing always an accelerated current in the direction 3, 5; an eddy will be formed from 3 to 4 and a portion of the flood passing over to 6, not unfrequently causes a smaller eddy from 6 to 7; in favoring situations the eddy from 3 to 4, appears sometimes to rival the strength and velocity of the principal stream: dangerous whirlpools are frequently produced in the situation w, occasioned by the counter currents; such a one exists at the grand gulf in the Mississippi, and in many other situations: we have seen one of about 5 feet diameter and 3 feet deep; all floating bodies passing within a certain distance of the vortex are attracted by it, and if not too large and buoyant, are precipitated to the bottom of the river, rising at the distance of 50, 100 or more yards from the place of descent: this imaginary energy of deep rivers, the result only of the descending fluid will nevertheless be extinguished as soon as the declivity of the surface is lost; rivers running a long course through an alluvial country, without the influx of auxiliary streams, are liable to stagnate before their junction with the ocean; the Nile is a remarkable example of this kind: and even the Mississippi, although we have said in general that it rolls a great body of water into the ocean, yet there has been at least one very extraordinary season, when the waters were sunken so uncommonly low, that there was no sensible current some distance within the mouth of the river. I have lately procured the following curious information from an intelligent *Gentleman of New Orleans who writes as follows, "In the beginning of "November 1800, when there was hardly any perceptible current in the Mississippi, I set off from the upper gate of the "city, in company with the master of a vessel, and sounded "the river at every three or four boats length until we landed "at the opposite shore: the depth of water increased pretty re-"gularly, viz. 10, 12, 13, 15, 17, 19 and 20 fathoms, the "greatest depth was found about 120 yards from the shore. "This operation was accurately performed, and as the river ri-"ses about 12 feet at this place, the depth at high water will "be 22 fathoms. A gentleman informed me that his father, "who was chief pilot in the time of the French, has often said "that a little way below the English Turn there was 50 fathoms, "and about the upper Plaquemine 60 fathoms. A respectable "inhabitant living six leagues below New Orleans, informed "me that during the above mentioned low state of the river, "the water was there found so brackish, that recourse was had "to the wells for drinking water, and that abundance of por-"poises, shark, and other sea fish were seen still higher up the "river. Many people thought the water brackish opposite to "the town. It had a greenish appearance, and when taken "up was very clear; and although I did not think it brackish, "I found it vapid and disagreeable." From the above curious relation it appears, that the waters of one of the greatest rivers on the globe were so completely dissipated that all current ceased 20 leagues above its mouth, nay the waters of the ocean flowed in (as into the Mediterranean) in order to restore the general level of waters. During the same period at Natchez, 380 miles from the mouth, the river flowed with a regular though very gentle current, (perhaps $\frac{1}{2}$ mile per hour) and a depth of 10 or 12 fathoms under the principal filament. What became of this great body of water? evaporation from

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the limited surface of the river is insufficient to account for so great a dissipation, but we know that the spongy texture of the alluvial soil is remarkably pervious to the waters of the river: from the flat and humid surface of the Delta, a perpetual evaporation exists, the lateral pressure of the waters of the river must supply the waste by exhalation, and this immense expence of fresh water, is to be accounted for by filtration and evaporation.

No. XXXIII.

Demonstration of a Geometrical Theorem; by Joseph Clay Esq. of Philadelphia.

Read July 20th, 1804.

THE following proposition was mentioned to me, some years since, as one which had been proposed by Mr. Simpson some time before his death. I do not know that any demonstration has hitherto been published.

From the angles at the base of any triangle, let two right lines be drawn cutting each other in any point within the triangle, and cutting the sides of the triangle, the segments of the sides and of the lines so drawn will form a trapezium; draw and bisect the diagonals, the right line joining the points of bisection, will, if produced, bisect the base of the triangle.

In the triangle ABC, (Fig. 6, Plate V.) draw CD, BE, cutting each other in F, and the sides of the triangle E and D. Draw AF and DE, and bisect them in G and H; draw GH, which if produced, will bisect the base of the triangle in K, making BK equal to KC.

Through F, draw LFM, NFO, parallel to AB and AC cutting the sides in M and O and the base in L and N: now because of the similar triangles, as CF is to CD so is FL to BD and LM to AB. Therefore by alternation as FL is to LM so is BD to AB. But as FL is to LM so is FN to CM; Therefore as BD is to AB so is FN to CM and the rectangle under BD, CM is equal to the rectangle under AB, FN. Again, as BF